

## EFFICIENT LABELING ALGORITHMS FOR ADJACENT QUADRATIC SHORTEST PATHS

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### ABSTRACT

In this article, we study the Adjacent Quadratic Shortest Path Problem (AQSP), which consists in finding the shortest path on a directed graph when its total weight component also includes the impact of consecutive arcs. We provide a formal description of the AQSP and propose an extension of Dijkstra's algorithm (that we denote aqD) for solving AQSPs in polynomial-time and provide a proof for its correctness under some mild assumptions. Furthermore, we introduce an adjacent quadratic A\* algorithm (that we denote aqA\*) with a backward search for cost-to-go estimation to speed up the search. We assess the performance of both algorithms by comparing their relative performance with benchmark algorithms from the scientific literature and carry out a thorough collection of sensitivity analysis of the methods on a set of problem characteristics using randomly generated graphs. Numerical results suggest that: (i) aqA\* outperforms all other algorithms, with a performance being about 75 times faster than aqD and the fastest alternative; (ii) the proposed solution procedures do not lose efficiency when the magnitude of quadratic costs vary; (iii) aqA\* and aqD are fastest on random graph instances, compared with benchmark algorithms from scientific literature. We conclude the numerical experiments by presenting a stress test of the AQSP in the context of real grid graph instances, with sizes up to  $16 \times 106$  nodes,  $64 \times 106$  arcs, and 109 quadratic arcs.

**Keywords:** Shortest Path Problem · Adjacent Quadratic Shortest Path Problem · Backward Cost-to-Go Estimation ·  $\alpha$ -Cycle · Spatial Routing

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